

REMINISCENCES OF WORKING WITH RAOUL BOTT

LORING W. TU

Even before meeting Raoul Bott in person at Harvard, I had already stumbled across his work, by accident, in my undergraduate days. My alma mater Princeton University prides itself on making its undergraduates do independent projects—a junior paper or seminar and a senior thesis, on topics not ordinarily covered in the curriculum. Depending on the student and the topic, this could be heaven or hell.

In my ignorance, I chose the Atiyah-Singer index theorem as my junior project, based on the hearsay of some graduate students that this was the most exciting development in all of mathematics in the preceding decade. For me, it soon turned into hell, for I could not make heads or tails of this undoubtedly exciting and powerful theorem. Searching for a graceful exit, I tried K -theory, which lies at the foundation of the index theorem. Even that was too much. I retreated further, to the foundation of K -theory, and there I found my salvation in the Bott periodicity theorem.

It is a hallmark of Raoul Bott's work that many of his theorems have an elegant, easily comprehensible statement, the periodicity theorem being a prime example. The proof, using either Morse theory or a little K -theory, could be mastered in a semester. It was the perfect junior-year project.

Later, upon entering Harvard for graduate study, I was immediately captivated by Bott's lectures. He really knew how to make mathematics come alive! In his hands, complicated constructions like the spectral sequence somehow became natural and manageable. I felt that I was not so much learning facts as witnessing the creation of mathematics. He taught in the Socratic method, pausing periodically to ask the students questions. He had a way of spicing up the lectures with colorful expressions that kept me wondering what was to come next. I remember well the class on curvature titled "Match Your Wits with Gauss" and many classes on the Čech-de Rham complex in which, he said, we were playing a giant "tic-tac-toe game."

As a graduate student, I assiduously attended all except one of Bott's courses, the one exception being too advanced for me at the time. After

Date: May 6, 2009.

getting a job at Tufts, only two subway stops from Harvard, I went back and sat in on a few more of Bott's courses. This meant that I repeated some classes, but like certain Chinese dishes that taste even better the next day, sometimes the second time around is better.

In 1998 at a conference to celebrate Victor Guillemin's sixtieth birthday, Guillemin proudly announced that he had taken twelve courses from Raoul Bott. That night, when I got home, I listed the courses of Raoul Bott I had attended, was attending, or would be attending. Much to my chagrin, I counted up only eleven. Since Bott has now retired from teaching, it looks like Guillemin will forever hold the world's record.

In spite of my devotion to Bott's courses, I do not have the distinction of being one of his Ph.D. students. When it came time to decide on a thesis advisor, I gravitated toward algebraic geometry and Phil Griffiths. It was in part because among the graduate students then, topology had the reputation of being a "dying field," where the only problems left were too hard to tackle, like the Poincaré conjecture. This impression was reinforced by the exodus to other fields of many people who had done great work in topology, people like Serre, Mazur, Hirzebruch, Smale, and Milnor.

I had attended a full-year course of Griffiths on algebraic geometry and had enjoyed the big book he was writing with Joe Harris based on the course. The presence of an algebraic structure on manifolds leads to myriad intricate relations that fascinated me. I became a Griffiths student.

In my fourth year at Harvard, Raoul Bott asked me if I would be interested in turning his course on algebraic topology into a book. Honored by the request, I readily agreed. It was a new venture for both of us and neither of us anticipated the amount of time it would end up taking.

Collaborating with Bott was an intellectual feast, but getting the manuscript typed was a nightmare. This was before the days of \TeX . To type a symbol, a typist had to change a ball in the typewriter; making an integral sign required a stencil. We went through many technical typists of varying quality, some of whom I had found through ads tacked to telephone poles in Cambridge. It seemed that the only qualification for advertising as a "technical typist" was the possession of an IBM symbol ball, for some technical typists I hired turned out to have never typed mathematics and did not know the Greek alphabet!

It happens often that an advisor has several students, but it is rare for a student to have several advisors at the same time. I was one of the few lucky ones. While working on a Ph.D. thesis under Phil Griffiths,

I continued my close association with Raoul Bott, attending his classes and working on the book with him. I must admit, at times I felt conflicted. My friend Giuseppe Canuto, also a Griffiths student, used to question my loyalty to my thesis advisor, calling me a “bigamist.”

Raoul and I spent countless hours discussing the book manuscript as we worked on it, sometimes in his office, sometimes at Dunster House, where he was the Master, and even on Martha’s Vineyard, where he had bought a summer house. He loved the sun and the beach. The beach near his house on the Vineyard had two sections, one which the locals called the “textile beach” for those with modesty, and the other, the nudist part, for the naturalists. During my visit, Raoul and his wife Phyllis took me to the textile beach. But he plucked the shoulder strap of Phyllis’ swimsuit and said, “Look. No tan line. Bad Mommy.” So I knew which part of the beach they usually went to.

I am by nature taciturn, but I’ve always found it easy to talk to Raoul. I often walk away from a conversation with him feeling particularly witty. When I think about other social occasions where he was not present, then it becomes clear the source of the sparkle and the wit.

My feeling of a special affinity with Raoul perhaps stems in part from the many coincidences in our background. Like him, in the formative years of my youth I left a relatively privileged background in my native land to come to this new world. Without realizing it, I even tracked his sojourns at McGill, Princeton, Harvard, and Michigan, most of the time as a penniless student. Upon graduation with a B.A., like him I briefly toyed with the idea of entering a medical school. However, these may only be surface similarities. What attracts me the most in Raoul is the constant sense of adventure and the joy he exudes in his own life and imparts to the lives of those around him.

For apart from insight into geometry and topology, I have also profited from Raoul’s practical advice. Once as we were returning to Boston from a conference, Raoul recounted to me several of his brushes with the law. The first went like this. One summer he did a house exchange with someone in Del Mar, California, near San Diego. The exchange included the use of a car. When Raoul, his wife, four children, and a dog flew into San Diego, they found that the California owner had left the key at the airline counter but had forgotten to say where to find the car. Raoul knew only that it was a Ford station wagon. Assuming that the owner had left in a hurry with the car parked close by, Raoul used the key to try to open all the Ford station wagons in the lot near the terminal. After much trial and error, he found the car. It was difficult to open the trunk, though, just like Raoul’s own car in Massachusetts.

Raoul thought to himself, "This is an even exchange, one bad car for another. Surely he cannot complain about my car." Each day the car became harder and harder to start. After one week, during a shopping expedition, it refused to start at all. It was then that Phyllis saw the name on the registration hanging from the dashboard, and it did not match that of the house owner. They had stolen a car for a week!

In the excitement of this discovery, the car suddenly started and they were able to drive home. The Botts had an emergency family meeting to discuss what to do. Some of the children suggested that Raoul should "drive the car somewhere and leave it." In the end Raoul called the police to explain the situation. After checking the files, the police said, "Yep. You are a car thief." Then the police said something that would strike terror in the heart of every law-abiding citizen: "You have been driving without insurance for a week. You're not covered by your insurance company or the car owner's. Don't touch that car again."

Shortly thereafter Raoul met the true owner and heard the other side of the story. The poor man had gone through every car in the airport lot, searching for his station wagon. In the process he found many cars with unlocked doors, some even with their keys hanging in the ignition. He was greatly puzzled: "With so many cars ripe for the picking, why did they take mine? Was it a desperate getaway? Someone on drugs?" The car owner added, "I didn't think of a Harvard professor."

Raoul's other encounters with the law were not nearly so dramatic and have now faded in my memory, but I remember clearly his parting advice to me that day, "If you are ever stopped by the police, make sure to say 'Officer' as early and as often as possible."

Sure enough, the next morning while zipping through Harvard Square in my car, I was stopped by a policeman. With the lesson fresh in my mind, I rolled down the window and asked, "Officer, what's the matter?" He said I did not observe a yield sign. "I'm sorry I didn't see it, Officer." I worked in the word "Officer" at least three times, to no avail. The policeman took my license and wrote up a ticket. I was too dejected to look at it, for in Massachusetts each moving violation results in a much higher insurance premium and it takes six years of spotless driving to erase the record. I was just about to erase a previous violation from five years ago. That night when I regained my

composure to read the ticket, I found that it was not a ticket, but a warning of no real consequence. Perhaps Raoul's advice worked.

DEPARTMENT OF MATHEMATICS, TUFTS UNIVERSITY, MEDFORD, MA 02155-7049

E-mail address: `ltu@tufts.edu`